Notre Dame University Bangladesh

Department of Computer Science & Engineering

Class Test – Artificial Intelligence

Full Marks: 30 **Time:** 60 minutes

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1. **[8 marks]** You roll two fair dice.

   (a) Compute the probability that the sum equals 7.

   (b) Compute the probability that the sum equals 12.

   (c) Are the two events "sum = 7" and "first die = 4" independent? Justify mathematically.

2. **[6 marks]** A spam filter uses the presence of the word **"discount"** to classify an email.

   - 70% of emails are not spam.
   - *40% of spam emails contain* "discount".
   - *5% of non-spam emails contain* "discount".

   Compute the probability that an email is spam given that it contains "discount". Show *each step using* **Bayes' Theorem.**

3. **[8 marks]** Consider a simple Bayesian Network with three variables:

   - R: Rain (true/false)
   - S: Sprinkler (true/false)
   - W: WetGrass (true/false)

   Given:

$$P(R) = 0.3, \quad P(S|R) = \langle 0.1, 0.5 \rangle, \quad P(W|S,R) = \begin{cases} 0.99 & \text{if } S, R \\ 0.90 & \text{if } S, \neg R \\ 0.80 & \text{if } \neg S, R \\ 0.00 & \text{if } \neg S, \neg R \end{cases}$$

Compute:

(a) $P(W, S, R)$

(b) $P(W = \text{true})$ using marginalization.

4. **Inference & Sampling (8 marks)**

Given the following joint probabilities:

Rain	Maintenance	Train	$P(R, M, T)$
none	yes	on time	0.224
none	no	on time	0.378
light	yes	delayed	0.016
light	no	on time	0.112
heavy	yes	delayed	0.024
heavy	no	delayed	0.036

(a) Estimate $P(\text{Train} = \text{delayed})$ using direct enumeration.

(b) Describe how you could estimate the same probability using *rejection sampling*.

(c) Explain in one or two lines why likelihood weighting is better than rejection sampling for rare evidence.

Chap-2

$P(\text{sum to } 12) = \frac{1}{36}$

$P(\text{ '' '' } 7) = \frac{6}{36} = \frac{1}{6}$

A = sum = 7

B = first die = 4

$\therefore P(A \cap B) = P(A)P(B) \rightarrow$ if independent.

$P(A) = \frac{6}{36} = \frac{1}{6}$

$P(B) = \frac{6}{36} = \frac{1}{6}$ [1st die = 4 & 2nd die can b
anything from 1-6]

$\therefore P(A \cap B)$ = only one probability $(4, 3) = \frac{1}{36}$

$P(A)P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ $\therefore$ They're indepe

Bayes rule -

$$P(b|a) = \frac{P(b)\,P(a|b)}{P(a)}$$

2) $P(N) = 0.7 \quad \therefore P(S) = 0.3$

$P(D|S) = 0.4 \quad P(D|N) = 0.05$

$\therefore P(S|D) = ?$

$$P(S|D) = \frac{P(S)\,P(D|S)}{P(S)\,P(D|S) + P(N)\,P(D|N)}$$

$$= \frac{0.3 \times 0.4}{(0.3 \times 0.4) + (0.7 \times 0.05)}$$

$= 77.4\%$

$P(S|R) = 0.1$
$P(S|\neg R) = 0.5$

3) Given, $P(R) = 0.3$; $P(S|R) = \langle 0.1, 0.5 \rangle$

$P(W|S,R) =$

Sprinkler	Rain	W	¬W
¬S	¬R	0.00	1
¬S	R	0.80	0.2
S	¬R	0.90	0.1
S	R	0.99	0.01

$\left[P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)\,P(B) \right]$

a) $P(W, S, R) = ?$

$P(W \cap S \cap R) = P(W|S,R) \times P(S|R) \times P(R)$

$= (0.99 \times 0.1 \times 0.3)$

$= 0.0297$

$P(R) = ? \qquad P(\neg R)$

b) $P(W=T) = P(W=T|S,R)\,P(S|R)\,P(R)$

1) ~~S|R~~ $\neg S \mid \neg R$		
2) $\neg S \mid R$		
3) $S \mid \neg R$		
4) $S \mid R$		

1) $P(W=T \mid \neg S \mid \neg R)\,\cancel{P(S|R)}\,P(\neg S \mid \neg R)\,P(\neg R)$

2) $P(W \mid \neg S \mid R) = P(\neg S \mid R)\,P(R)$

3) $P(W \mid S \mid \neg R)\,P(S \mid \neg R)\,P(\neg R)$

4) $P(W \mid S \mid R)\,P(S \mid R)\,P(R)$

Final marginalization = 1+2+3+4

~~4) a) P(light rain:~~

4) a) P(light | yes | delayed) = 0.016

P(heavy | yes | delayed) = 0.024

P(heavey | no | delayed) = 0.036

P(Train = delayed) = 0.016 + 0.024 + 0.036

= 0.076

b) Generate ~~mar~~ all the samples from the prior joint distribution, then keep only the samples where the event matches the condition we are estimating.

→ count how many generated samples have train = delayed.

c) Rejection sampling wastes most samples when evidence is rare because almost all generated samples get discarded.

On the other hand, likelihood weighting keeps all samples and adjusts them using weights, so it is better.